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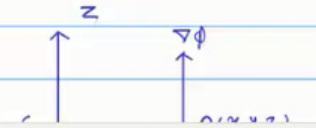
File Edit View Navigation Journal Tools Plugin Help

$d\phi = \nabla\phi \cdot d\vec{r} = |\nabla\phi| |d\vec{r}| \cos\theta$ , where  $\theta$  is the angle between the vectors  $\nabla\phi$  and  $d\vec{r}$ . If  $\nabla\phi$  and  $d\vec{r}$  are in the same direction then  $\theta = 0 \therefore d\phi = |\nabla\phi| |d\vec{r}|$

Since  $\cos\theta = 1$  is the maximum value of  $\cos\theta$ ,  $d\phi$  is maximum at  $\theta = 0$ , and its maximum value is  $|\nabla\phi| |d\vec{r}|$

Applications of grad $\phi$  ( $\nabla\phi$ ):

1) Normal:



Double-click to go to full screen, ctrl+click to snap to video size

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01:17



Microsoft Excel ribbon showing tabs: Home, Insert, Page Layout, Formulas, Data, Review, View. The View tab is active, displaying options for Ruler, Formula Bar, Gridlines, Headings, Message Bar, Zoom (100%), and various window management tools like Split, Hide, and Synchronous Scrolling.

Session ID	Topic	Meeting T	Username	Name	Date	Duration	Invited	Registered	Client atte	Audio typ	People Mi	Local toll c	Local toll-f	Local callo	Intl. callou	Internal C	Intl. call-in	Integrated VoIP
1.85192E+17	SH_226	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	02-03-2021	98 mins	0	N/A	86	VoIP	3785	0	0	0	0	0	0	3727
1.85112E+17	S D Ahirrao's Personal	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	02-02-2021	1 min	0	N/A	1	VoIP	1	0	0	0	0	0	0	1
1.84576E+17	S D Ahirrao's Personal	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	02-02-2021	1 min	0	N/A	1	VoIP	1	0	0	0	0	0	0	1
1.85103E+17	sh226_020221	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	02-02-2021	91 mins	0	N/A	83	VoIP	3391	0	0	0	0	0	0	3357
1.85013E+17	sh226	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	02-01-2021	76 mins	0	N/A	61	VoIP	2579	0	0	0	0	0	0	2511
1.84831E+17	sh226300121	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	1/30/2021	90 mins	0	N/A	51	VoIP	2051	0	0	0	0	0	0	2032
1.84739E+17	sh226	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	1/29/2021	102 mins	0	N/A	38	VoIP	2051	0	0	0	0	0	0	2046
1.84649E+17	sh226	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	1/28/2021	106 mins	0	N/A	40	VoIP	1845	0	0	0	0	0	0	1821
1.84574E+17	sh226	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	1/27/2021	44 mins	0	N/A	21	VoIP	603	0	0	0	0	0	0	590
1.84573E+17	SH226:Engineering M	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	1/27/2021	14 mins	28	N/A	1	VoIP	14	0	0	0	0	0	0	14
1.84572E+17	SH226:Engineering M	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	1/27/2021	3 mins	28	N/A	1	VoIP	3	0	0	0	0	0	0	3
1.84572E+17	sh226	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	1/27/2021	13 mins	0	N/A	10	VoIP	77	0	0	0	0	0	0	76
1.84567E+17	SH226:Engineering M	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	1/27/2021	23 mins	28	N/A	1	VoIP	23	0	0	0	0	0	0	22
1.84566E+17	S D Ahirrao's Personal	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	1/27/2021	37 mins	0	N/A	31	VoIP	408	0	0	0	0	0	0	386
1.84433E+17	S D Ahirrao's Personal	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	1/27/2021	1 min	0	N/A	1	VoIP	1	0	0	0	0	0	0	0
1.84144E+17	SH226:Engineering M	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	1/27/2021	17 mins	28	N/A	1	VoIP	17	0	0	0	0	0	0	0
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1.8443E+17	sh226	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	1/25/2021	15 mins	0	N/A	4	VoIP	25	0	0	0	0	0	0	23
1.84385E+17	SH226 ENGINEERING	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	1/25/2021	51 mins	0	N/A	4	VoIP	44	0	0	0	0	0	0	44
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1.84144E+17	SH226:Engineering M	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	1/24/2021	23 mins	30	N/A	1	VoIP	23	0	0	0	0	0	0	23
1.84233E+17	trial meeting	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	1/23/2021	1 min	1	N/A	1	VoIP	1	0	0	0	0	0	0	1
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1.84205E+17	SH226:Engineering M	Meetings	sdahirrao@gcoej.ac.in	S D Ahirrao	1/23/2021	20 mins	0	N/A	1	VoIP	20	0	0	0	0	0	0	20

# APPLICATION OF LDE TO ELECTRICAL CIRCUIT

## Kirchhoff's Voltage Law (KVL) :

The algebraic sum of the voltage drops around any closed circuit is equal to the total electromotive force (e.m.f.) in the circuit

## Kirchhoff's Current Law (KCL) :

At any point of a circuit the sum of the inflowing currents is equal to the sum of the out flowing current.

*We consider the following circuits.*

## i) L – C circuit

We consider the following L - C circuits without and with an electromotive force  $E(t)$ .

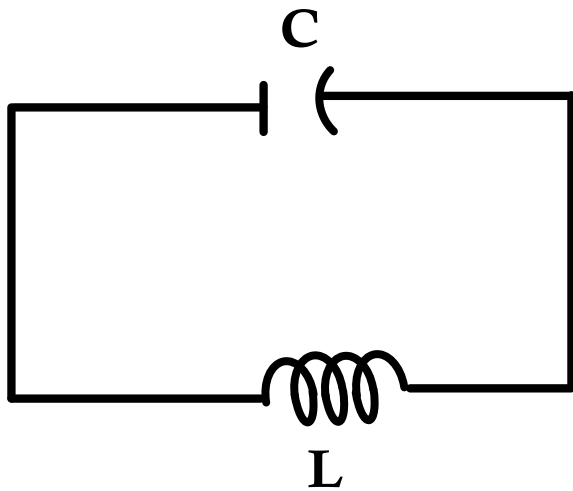


Figure ( a )

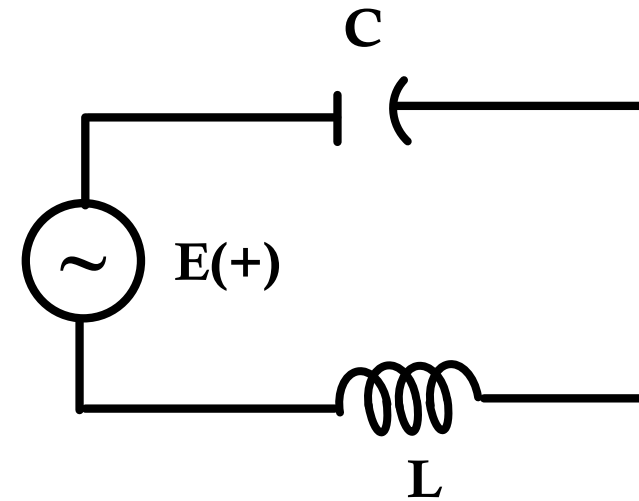


Figure ( b )



The voltage drop across inductor =  $L \frac{di}{dt} = L \frac{d^2q}{dt^2}$

$$\left[ \because i = \frac{dq}{dt} \right]$$

The voltage drop across capacitor =  $\frac{q}{C}$

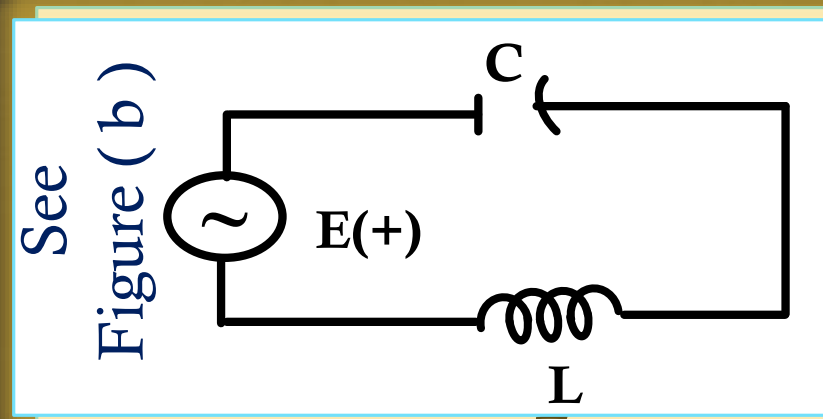
By Kirchhoff's voltage law, the sum of voltage drop across the capacitor and inductor is equals to total e.m.f. in the circuit.

Thus the differential equation for the circuit in fig ( a ) and fig ( b ) respectively are

$$L \frac{d^2q}{dt^2} + \frac{q}{C} = 0 \quad \Rightarrow \quad \frac{d^2q}{dt^2} + \frac{1}{LC} q = 0$$

and

$$L \frac{d^2q}{dt^2} + \frac{q}{C} = E(t) \Rightarrow \frac{d^2q}{dt^2} + \frac{1}{LC} q = \frac{1}{L} E(t)$$



### i) L – C – R Circuits :

Consider the following L – C – R circuits without and with an electromotive force  $E(t)$ .

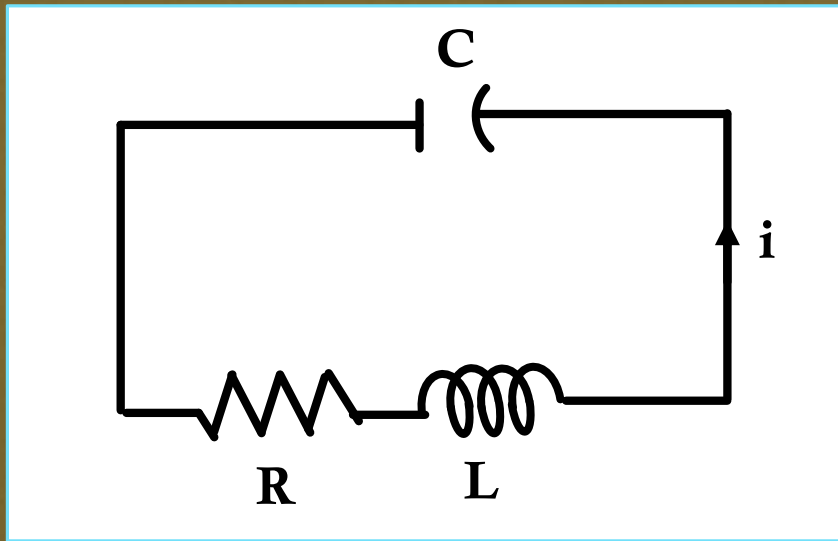


Figure ( c )

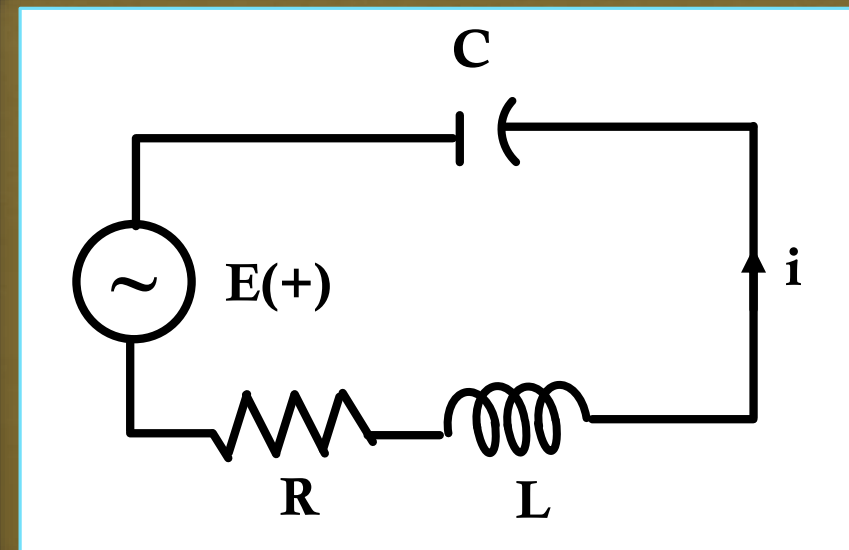


Figure ( d )

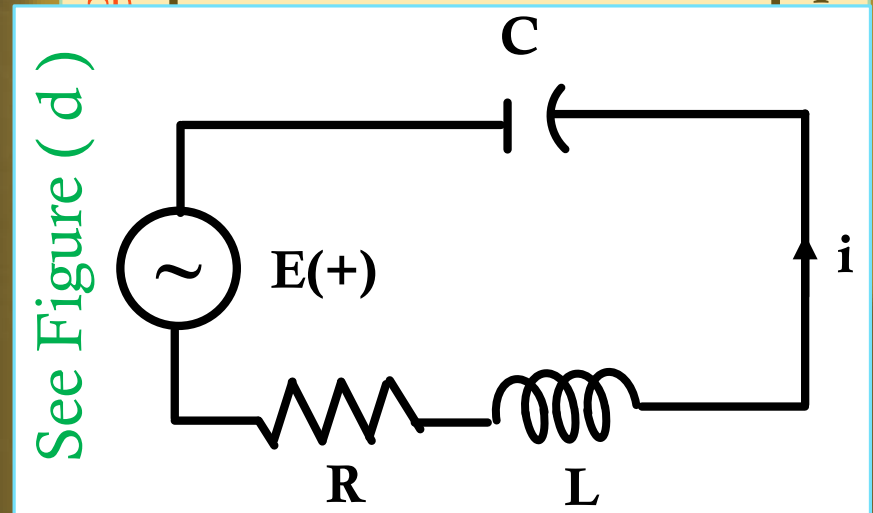
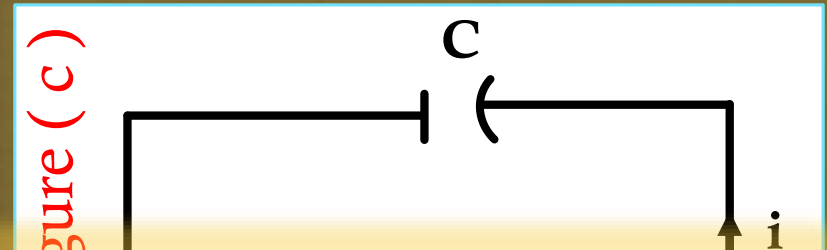
Since the voltage drop across  $L$ ,  $C$  and  $R$  respectively.

$$L \frac{di}{dt} = L \frac{d^2q}{dt^2}, \quad \frac{q}{C} \quad \text{and} \quad R_i = R \frac{dq}{dt}$$

Therefore by **Kirchhoff's voltage law**, differential equation for the circuits in fig. (c) and fig. (d) are respectively.

$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = 0$$

$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = E(t) \quad \text{--- --- --- (i)}$$



Note that the differential equation ( i ) contains dependent variable  $q$ .

Differentiating ( i ) w.r.t.  $t$  we get

$$L \frac{d^3 q}{dt^3} + R \frac{d^2 q}{dt^2} + \frac{1}{C} \frac{dq}{dt} = \frac{dE}{dt} \quad \text{--- --- --- ( ii )}$$

Using  $i = \frac{dq}{dt}$ , in ( ii ) we get

$$L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{i}{C} = \frac{dE}{dt} \quad \text{--- --- --- ( iii )}$$

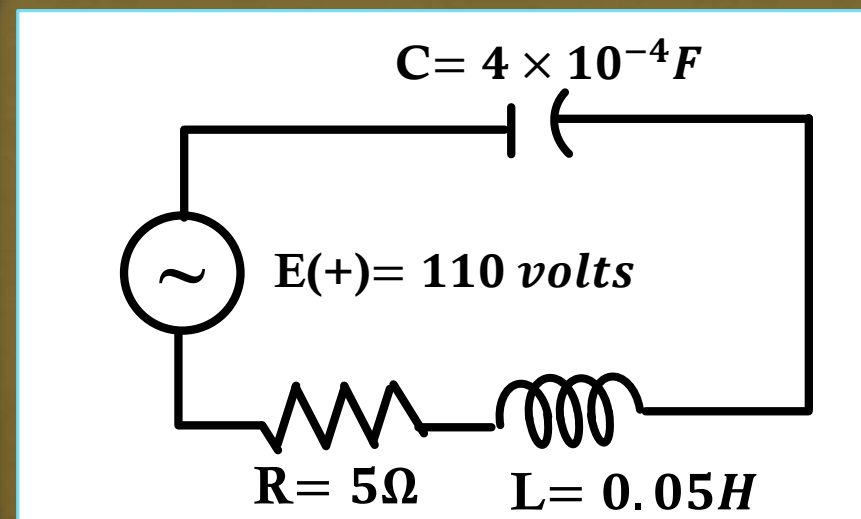
Thus the charge  $q$  and current  $i$  at any time in L – C – R circuits are obtained from ( ii ) and ( iii ) which are **second order linear differential equation with constant coefficients**.

On the same line we can obtain the differential equation in the form of dependent variable  $i$ , for the **L – C circuits** given in figure (a) and figure (b)



**Example** :- A circuit consist of an inductance of  $0.05$  henrys , a resistance  $5$  ohms and condenser of capacitance  $4 \times 10^{-4}$  farad . If  $q = i = 0$  when  $t = 0$ , find  $q$  and  $i$  when there is a constant e.m.f. of  $110$  volts.

**Solution** :-



We know the voltage drop across  $L$  ,  $C$  and  $R$  are respectively

$$L = \frac{d^2q}{dt^2}, \frac{q}{C} \quad \text{and} \quad R \frac{dq}{dt}$$

Thus by Kirchhoff's voltage law, we have

$$L \frac{d^2i}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = E(t) \quad \text{--- --- --- (i)}$$

Here  $L = 0.05 \text{ H}$ ,  $R = 5 \Omega$ ,  $C = 4 \times 10^{-4} \text{ F}$  and  $E(t) = 110 \text{ volts}$

Thus the equation (i) becomes

$$0.05 \frac{d^2q}{dt^2} + 5 \frac{dq}{dt} + \frac{q}{4 \times 10^{-4}} = 110$$

$$\Rightarrow \frac{d^2q}{dt^2} + 100 \frac{dq}{dt} + 50000q = 20(110) = 2200 \quad \text{--- --- --- (ii)}$$

Let  $\frac{d}{dt} \equiv D$ , then equation (ii) can be written as  $(D^2 + 100D + 50000)q = 2200$

The A.E. is  $D^2 + 100D + 50000 = 0$

$$D = \frac{-100 \pm \sqrt{(100)^2 - 4(1)50000}}{2(1)}$$

$$= \frac{-100 \pm \sqrt{-19,0000}}{2}$$

$$= \frac{-100 \pm i 100\sqrt{19}}{2}$$

$$= -50 \pm i 50\sqrt{19}$$

$$C.F. = e^{-50t} [C_1 \cos(50\sqrt{19})t + C_2 \sin(50\sqrt{19})t]$$

$$\begin{aligned}\text{And } P.I. &= \frac{1}{D^2+100D+50000} 2200 \\ &= \frac{1}{D^2 + 100D + 50000} 2200 e^{0t} \\ &= \frac{1}{50000} 2200\end{aligned}$$

$$P.I = \frac{11}{250}$$

*The complete solution is*

$$q = C.F. + P.I$$

$$q = e^{-50t} [C_1 \cos(50\sqrt{19})t + C_2 \sin(50\sqrt{19})t] + \frac{11}{250} \quad \text{--- (iii)}$$

Differentiating Eq. ( iii ) w.r.t.  $t$  and using  $i = \frac{dq}{dt}$ , we get

$$i = e^{-50t} [-50\sqrt{19} C_1 \sin(50 \sqrt{19} t) + 50\sqrt{19} C_2 \cos(50 \sqrt{19} t)] + (-50)e^{-50t} [C_1 \cos(50 \sqrt{19} t) + C_2 \sin(50 \sqrt{19} t)] \quad \text{--- ( iv )}$$

Given  $q = 0$ ,  $i = 0$  when  $t = 0$

Thus from Eq. ( iii ), we get

$$q = e^{-50t} [C_1 \cos(50\sqrt{19})t + C_2 \sin(50\sqrt{19})t] + \frac{11}{250}$$

$$0 = C_1 + \frac{11}{250}, \quad C_1 = -\frac{11}{250} \quad \text{and from Eq. ( iv ) we get}$$

$$0 = 50 \sqrt{19} C_2 + C_1(-50)$$

$$= 50 \sqrt{19} C_2 + \left(-\frac{11}{250}\right)(-50) = \sqrt{19} C_2 + \frac{11}{250} \quad C_2 = -\frac{11}{250\sqrt{19}}$$



Thus From (iii) , we have

$$q = e^{-50t} [C_1 \cos(50\sqrt{19})t + C_2 \sin(50\sqrt{19})t] + \frac{11}{250}$$

$$q = e^{-50t} \left[ -\frac{11}{250} \cos(50\sqrt{19} t) - \frac{11}{250\sqrt{19}} \sin(50\sqrt{19} t) \right] + \frac{11}{250}$$

And From (iv) , we have

$$i = e^{-50t} [-50\sqrt{19} C_1 \sin(50\sqrt{19} t) + 50\sqrt{19} C_2 \cos(50\sqrt{19} t)] \\ + (-50)e^{-50t} [C_1 \cos(50\sqrt{19} t) + C_2 \sin(50\sqrt{19} t)]$$

$$i = e^{-50t} \left[ -\frac{11\sqrt{19}}{5} \sin(50\sqrt{19} t) - \frac{11}{5} \cos(50\sqrt{19} t) \right] + \\ e^{-50t} \left[ \frac{11}{5} \cos(50\sqrt{19} t) + \frac{11}{5\sqrt{19}} \sin(50\sqrt{19} t) \right]$$

**Example** :- An unchanged condenser of capacity  $C$  is charged by applying an e.m.f.  $E \sin\left(\frac{t}{\sqrt{LC}}\right)$ , through lead of self inductance  $L$  and negligible resistance. Prove that at time  $t$ , the charge on one of the plates is  $\frac{EC}{2} \left[ \sin\left(\frac{t}{\sqrt{LC}}\right) - \frac{t}{\sqrt{LC}} \cos\left(\frac{t}{\sqrt{LC}}\right) \right]$

**Solution** :- we know that voltage drop across condenser and inductor are  $L \frac{d^2q}{dt^2}$  and  $\frac{q}{C}$  respectively.

The total e.m.f. is given  $E \sin\left(\frac{t}{\sqrt{LC}}\right)$

$\therefore$  By **Kirchhoff's voltage law**, we have  $L \frac{d^2q}{dt^2} + \frac{q}{C} = E \sin\left(\frac{t}{\sqrt{LC}}\right)$

$$\therefore \frac{d^2q}{dt^2} + \frac{1}{LC} q = \frac{E}{L} \sin\left(\frac{t}{\sqrt{LC}}\right) \quad \text{--- (i)}$$

Let  $\frac{d}{dt} = D$ , then (i) becomes  $(D^2 + w^2)q = \frac{E}{L} \sin wt$ , where  $W = \frac{1}{\sqrt{LC}}$

The A.E. is  $D^2 + w^2 = 0 \Rightarrow D = \pm iw$

$$\therefore C.F. = C_1 \cos wt + C_2 \sin wt$$

$$\therefore P.I. = \frac{1}{D^2 + w^2} \frac{E}{L} \sin wt$$

$$= \frac{E}{L} \frac{t}{2D} \sin wt$$

$$P.I. = \frac{E}{L} \frac{t}{2} \left( -\frac{\cos wt}{w} \right)$$

The complete solution is

$$q = C.F. + P.I.$$

$$q = C_1 \cos wt + C_2 \sin wt - \frac{E}{2Lw} t \cos wt \quad \text{--- (ii)}$$

$$i = \frac{dq}{dt}$$

$$= -C_1 w \sin wt + C_2 w \cos wt - \frac{E}{2Lw} [-t(w) \sin wt + \cos wt] \quad \text{--- (iii)}$$

Given that  $q = 0$  and  $i = 0$  at  $t = 0$ .

Thus from Eq. (ii) we have  $0 = C_1$  and from Eq.(iii) we have

$$0 = C_2 w - \frac{E}{2Lw} \Rightarrow C_2 = \frac{E}{2Lw^2}$$

The Eq. (ii) becomes

$$q = \frac{E}{2Lw^2} \sin wt - \frac{E}{2Lw} t \cos wt$$

$$= \frac{E}{2L\omega^2} [\sin \omega t - \omega t \cos \omega t]$$

As  $\omega = \frac{1}{\sqrt{LC}}$  ,  $\omega^2 = \frac{1}{LC}$  , we have

$$q = \frac{EC}{2} \left[ \sin \left( \frac{t}{\sqrt{LC}} \right) - \frac{t}{\sqrt{LC}} \cos \left( \frac{t}{\sqrt{LC}} \right) \right]$$



Regression: It is the method of estimating the value of one variable when that of the other is known and the variables are correlated.

Line of regression of  $y$  on  $x$  :: It is the line which gives the best estimate for the values of  $y$  for any given value of  $x$ .  
This regression equation of  $y$  on  $x$  is given by

$$y - \bar{y} = r \frac{s_y}{s_x} (x - \bar{x})$$

Where  $r$  is coefficient of correlation between the variables  $x, y$

$$s_x = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}$$

$$s_y = \sqrt{\frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n}}$$

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

$$\bar{y} = \frac{\sum_{i=1}^n y_i}{n}$$

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}}$$

the coefficient of correlation between  $x, y$

The line of regression of  $y$  on  $x$  also can be written as  
 $y = bx + c$

The line of regression of  $x$  on  $y$  is given by

$$x - \bar{x} = r \frac{s_x}{s_y} (y - \bar{y})$$

$(\bar{x}, \bar{y})$

It is the line which gives the best estimate for the values of  $x$  for any given value of  $y$ .

It is can be written as

$$x = a + by$$

In these regression equations  $\bar{x}$  and  $\bar{y}$  are means of the  $x$  series &  $y$  series respectively

The line regression is line of best fit and is obtained by the principle of least squares.

Regression coefficient of  $y$  on  $x$ :

The slope  $b$  of the line of regression of  $y$  on  $x$  is called as coefficient of regression of  $y$  on  $x$ . It represents the increment in the value of  $y$  corresponding to a unit change in the value of  $x$ .

It is represented by

$$b_{yx} = \text{Regression coefficient of } y \text{ on } x = r \frac{b_y}{b_x}$$

Similarly the slope  $b$  of <sup>line of</sup> regression of  $x$  on  $y$  is called the coefficient of regression of  $x$  on  $y$  and is represented by

$$b_{xy} = r \frac{b_x}{b_y}$$

$$* \quad b_{xy} \cdot b_{yx} = r^2$$

$$r = \pm \sqrt{b_{xy} \cdot b_{yx}}$$

$r$  will be negative if the both coefficients are negative

$r$  will be +ve if the both coefficients are positive

$$* \quad -1 \leq r \leq 1$$

Expressions for regression coeffs:

$$b_{yx} = r \frac{b_y}{b_x} = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

$$b_{xy} = r \frac{b_x}{b_y} = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (y - \bar{y})^2}$$

$$* \quad r = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sqrt{\sum x^2 - \frac{(\sum x)^2}{n}} \sqrt{\sum y^2 - \frac{(\sum y)^2}{n}}}, \quad b_x = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n}}, \quad b_y = \sqrt{\frac{\sum y^2 - \frac{(\sum y)^2}{n}}{n}}$$

$$b_{yx} = r \frac{b_y}{b_x} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}}$$

$$b_{xy} = r \frac{b_x}{b_y} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum y^2 - \frac{(\sum y)^2}{n}}$$

$$* \quad \text{If } b_{yx} > 1 \text{ then } b_{xy} < 1 \quad \text{If } r^2 \leq 1 \quad r^2 = b_{yx} b_{xy} \leq 1 \\ \Rightarrow b_{yx} \leq \frac{1}{b_{xy}}$$

Regression coefficients are independent of the change of the origin, but not of scale!

$$\rightarrow dx = \frac{x-a}{h}, \quad dy = \frac{y-b}{k}$$

$$x = a + hd, \quad y = b + kd, \quad a, b, h (>0) \text{ \& } k (>0) \text{ are constants}$$

$$r_{dxdy} = r_{xy}, \quad b_{yx} = \frac{1}{h^2} b_x^2, \quad b_{dy} = \frac{1}{k^2} b_y^2$$

$$b_{dydy} = r_{dxdy} \cdot \frac{b_{dy}}{b_{dx}} = \frac{k}{h} b_{yx}$$

$$b_{dydx} = \frac{h}{k} b_{yx}$$

\* Both regression coefficients will have same sign i.e. either both are positive or both are negative.

$$* \quad r > 0 \quad \text{if } b_{yx} > 0 \quad b_{yx} \geq 0$$

$$* \quad r < 0 \quad \text{if } b_{yx} < 0 \quad b_{yx} < 0$$

\*  $r = 0$ , these lines of regression are per to each other

? \* The lines of regressions pass through the point  $(\bar{x}, \bar{y})$

Ex: The regression lines of sample are  $x + 6y = 6$  &  $3x + 2y = 10$ .

Find i) Sample means  $\bar{x}$  and  $\bar{y}$

ii) The coefficient of correlation bet<sup>n</sup>  $x$  &  $y$

iii) Also estimate  $y$  when  $x = 12$

$\rightarrow$   $\because$  The lines of regression pass through the point  $(\bar{x}, \bar{y})$

$$\text{we have } \bar{x} + 6\bar{y} = 6 \quad \text{--- (I)}$$

$$3\bar{x} + 2\bar{y} = 10 \quad \text{--- (II)}$$

$$3 \times \text{eqn (I)} - \text{eqn (II)}$$

$$3\bar{x} + 18\bar{y} - 3\bar{x} - 2\bar{y} = 18 - 10$$

$$16\bar{y} = 8 \quad \therefore \bar{y} = \frac{1}{2}$$

$$\therefore \bar{x} = 6 - 6\bar{y} = 6 - \frac{6}{2} = 6 - 3 = 3$$

$$\bar{x} = 3, \quad \bar{y} = \frac{1}{2}$$

(ii) Let the  $x + 6y = 6$  be the line of regression of  $x$  on  $y$

$$x = -6y + 6$$

regression coefficient of  $x$  on  $y = -6 = b_{yx}$

Let the line regression of  $y$  on  $x$  be  $3x + 2y = 10$

$$\therefore 2y = -3x + 10$$

$$y = -\frac{3}{2}x + 5$$

$\therefore$  regression coefficient of  $y$  on  $x = b_{yx} = -\frac{3}{2}$

$$r^2 = b_{xy} b_{yx} = (-6) \left(-\frac{3}{2}\right) = 9$$

$$r = \pm 3 \quad \text{but } -1 \leq r \leq 1$$

Our choice of regression lines is wrong

So we change our choice of regression lines

Let the  $x + 6y = 6$  be the line of regression<sup>o</sup> -  $y$  on  $x$

$$y = -\frac{x}{6} + 1$$

regression coefficient of  $y$  on  $x = -\frac{1}{6} = b_{yx} < 0$

Let the  $3x + 2y = 10$  be the line of regression<sup>o</sup> -  $x$  on  $y$

$$x = -\frac{2}{3}y + \frac{10}{3}$$


regression coefficient of  $x$  on  $y = -\frac{2}{3} = b_{xy} < 0$

$$\text{Now } r^2 = b_{xy} b_{yx} = \left(-\frac{1}{6}\right) \left(-\frac{2}{3}\right) = \frac{2}{18} = \frac{1}{9}$$

$$r = \pm \frac{1}{3} \quad (\text{which is in the range } -1 \leq r \leq 1)$$

but  $b_{yx} < 0$ ,  $b_{xy} < 0$

$\therefore r = -\frac{1}{3}$  i.e.  $x, y$  are negatively linearly correlated.



**Instrumentation Engineering Department  
Government College of Engineering,  
Jalgaon**

**ONLINE CLASS I: DIGITAL IMAGE PROCESSING**

**Date: 11/08/2020**

**Course Teacher,  
Miss. PRIYANKA A KHARCHE**





Course Name

**DIGITAL IMAGE PROCESSING  
(IN401)**

# SYLLABUS:-

## Course Content

### Digital Image Fundamentals

Introduction to image processing, digital image representation, pixel, intensity, gray level, brightness, contrast, steps in image processing, elements of DIP system, elements of visual perception, simple image formation model, colour model, sampling and quantization, imaging geometry, 2D linear convolution, 2D circular convolution, 2D correlation.

### Image Transforms

Introduction to image transforms and its need, classification of image transform, 2D Discrete Fourier Transform and properties, FFT (Decimation in time and Decimation in frequency technique), Walsh Transform, Hadamard Transform, Discrete Cosine Transform, Haar Transform, Slant Transform, Wavelet Transform..

### Image Enhancement

Image enhancement by point processing operations- Histogram and histogram equalization, contrast stretching, thresholding, image negative, intensity level slicing, bit extraction, range compression, gamma correction, image arithmetic, Spatial filtering- Low-pass filter, weighted averaging filter, median filter, high-pass filter, Bartlett filter, Gaussian filter, Frequency domain methods- Low-pass and high-pass Butterworth filter, low-pass and high-pass Gaussian filter, homomorphic filter.

### Image Restoration and Image Segmentation

Image restoration-degradation model, Linear image restoration techniques, Non-linear image restoration techniques, Blind deconvolution, Image denoising.

Image segmentation- Region approach to image segmentation, clustering techniques, segmentation based on thresholding, edge-based segmentation, edge linking, Watershed transformation.

### Image Compression

Need for image compression, types of redundancies, Run length coding, Shannon-Fano coding, Huffman coding, Arithmetic coding, Predictive coding, Transform based compression, Image compression standards, Scalar and vector quantization.

### Text Books:

1. Digital Image Processing, R. C. Gonzalez and R. E. Woods, Pearson Education, 3<sup>rd</sup> edition, 2008.
2. Fundamentals of Digital Image Processing, Anil K. Jain, Prentice Hall India, 2010.
3. Digital Image Processing, S. Jayaraman, S. Esakkirajan and T. Veerakumar, McGraw Hill Publication, 2012.

### Reference Books:

1. Image Processing, Analysis and Machine Vision, M. Sonka, V. Hlavac and R. Boyle, Thomson Learning, 2<sup>nd</sup> edition, 2001.
2. Digital Image Processing, Kenneth Castleman, Pearson Education, 2006.



# UNIT I:- DIGITAL IMAGE FUNDAMENTALS

## **Digital Image Processing**

### **INTRODUCTION**

**Processing of images which are digital  
in nature by a Digital Computer**



- Why do we need Image Processing?

- It is motivated by two major applications-
  1. Improvement of pictorial information for human perception.
  2. Image Processing for autonomous machine application.
  3. Efficient storage and transmission

# Human Perception:-

- Employ methods capable of enhancing pictorial information for human interpretation and analysis:
  - Typical applications:
    - Noise Filtering
    - Content enhancement
      - i) Contrast Enhancement
      - ii) Deblurring
    - Remote Sensing



# Filtering:-



Noisy Image

4



Filtered Image

# Image Enhancement:-

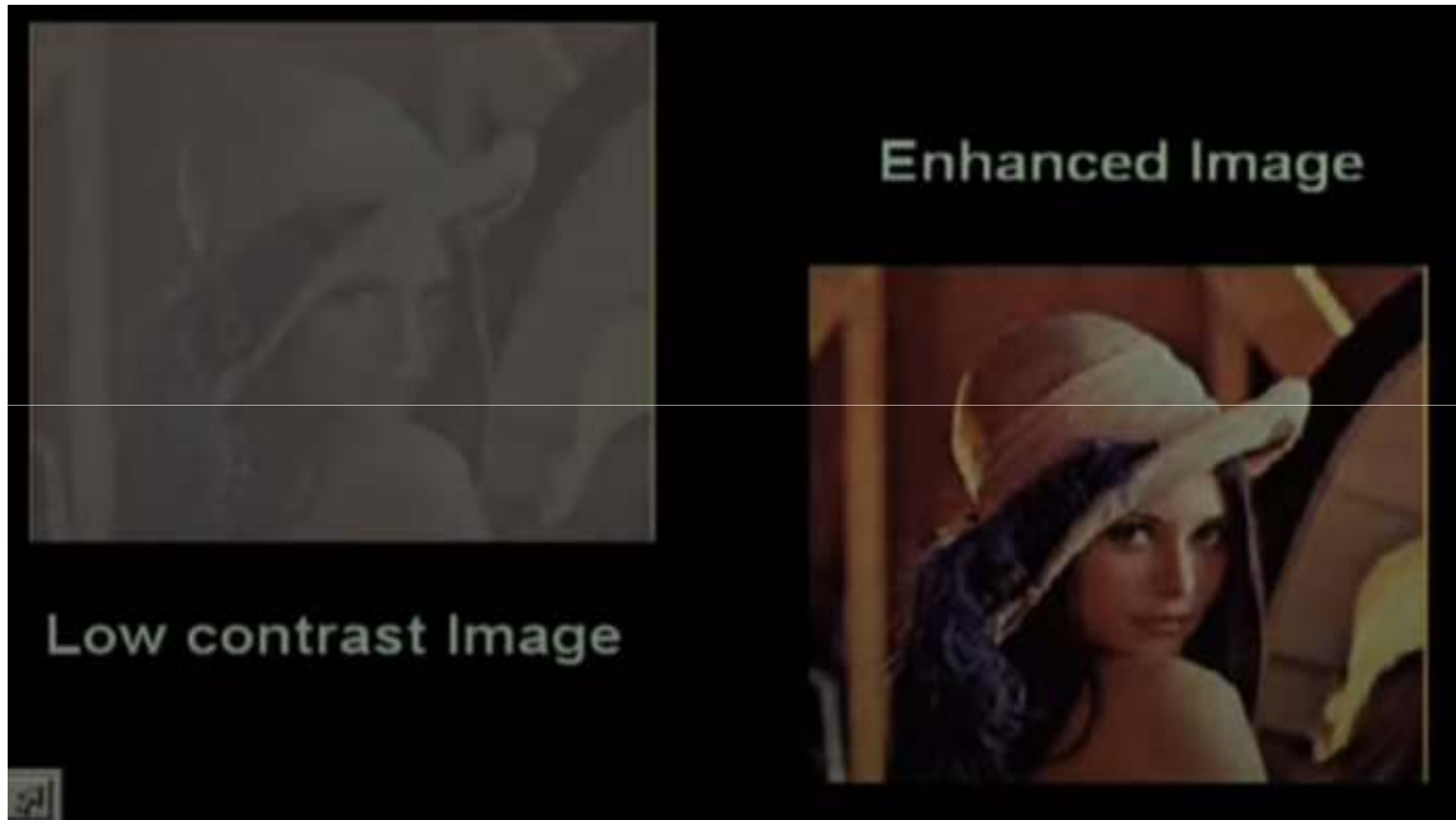


Low contrast Image

Enhanced Image



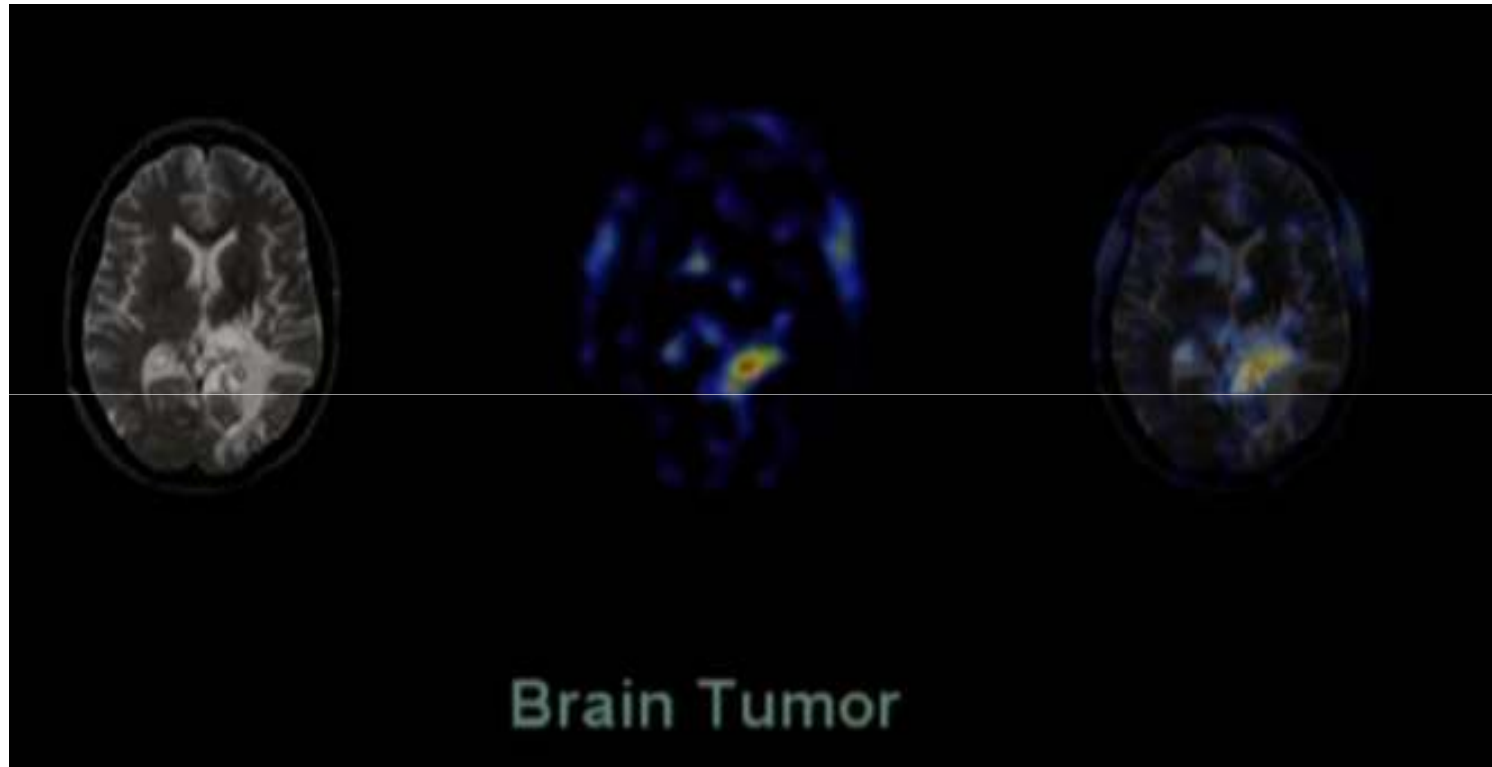
# Image Enhancement:-



# Image Deblurring:-



# Medical Imaging:-

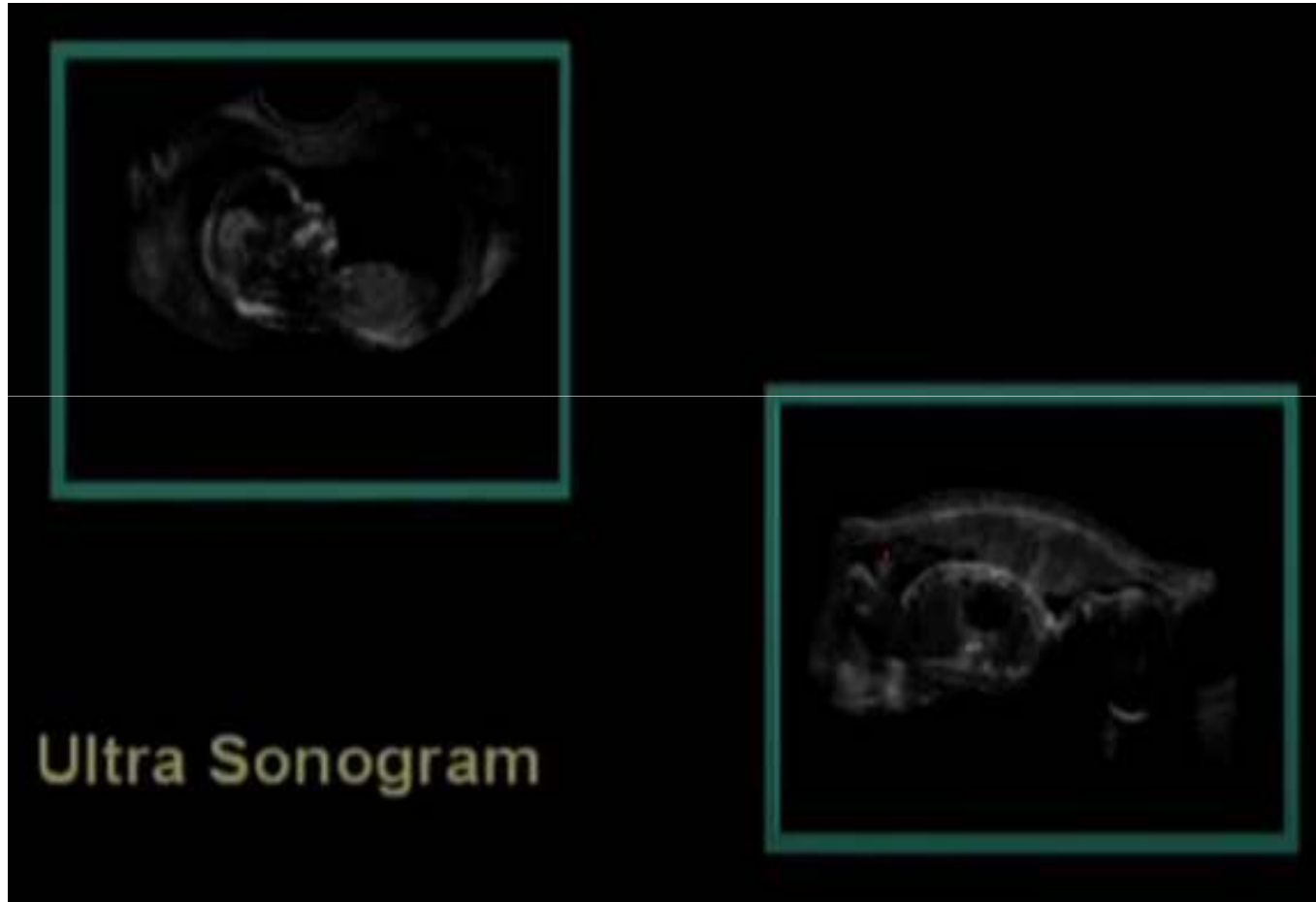


# Medical Imaging:-



Cancer Detection

# Medical Imaging:-



# Remote Sensing:-



# Remote Sensing:-








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# F Y Civil

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Sunil Ahirrao posted a new assignment: Audit course submission  
6 Aug 2021 (Edited 6 Aug 2021) 

Sunil Ahirrao posted a new assignment: Anti-ragging form registration and declaration  
8 Apr 2021 (Edited 8 Apr 2021) 





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GCOEJ: S. Y. B. Tech(2021)  
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Sunil Ahirrao posted a new assignment: complimentary MSE :SH296U (Advance ... ▶ 1 student  
24 Jul 2021



Sunil Ahirrao posted a new assignment: Surprise Test  
7 May 2021



Sunil Ahirrao posted a new assignment: MSE April 2021: S.Y.B.Tech.Civil (AEM)





Civil Engineering Department ICT Tools used proofs

Sr. No.	Topic	Youtube video lecture link
<b>Course : CE351U (ADVANCED THEORY OF STRUCTURES)</b>		
1	Basic Concepts of Structural Analysis	<a href="https://www.youtube.com/watch?v=JPFdrMftdao&amp;list=PLgWHRReJ5o_K3EEyJiLCY76erSoWUa7PBn">https://www.youtube.com/watch?v=JPFdrMftdao&amp;list=PLgWHRReJ5o_K3EEyJiLCY76erSoWUa7PBn</a> <a href="https://www.youtube.com/watch?v=x7HPZ5IZixE&amp;list=PLgWHRReJ5o_K3EEyJiLCY76erSoWUa7PBn&amp;index=2">https://www.youtube.com/watch?v=x7HPZ5IZixE&amp;list=PLgWHRReJ5o_K3EEyJiLCY76erSoWUa7PBn&amp;index=2</a>
2	Slope Deflection Method	<a href="https://www.youtube.com/watch?v=gTXQ35j1Qoo&amp;list=PLgWHRReJ5o_K3xOmVQYNLgy1y39apzeozt">https://www.youtube.com/watch?v=gTXQ35j1Qoo&amp;list=PLgWHRReJ5o_K3xOmVQYNLgy1y39apzeozt</a>
3	Moment Distribution Method	<a href="https://www.youtube.com/watch?v=09BmYDmyHO0&amp;list=PLgWHRReJ5o_K35M3Fme9fkqjIij0WMO2SA">https://www.youtube.com/watch?v=09BmYDmyHO0&amp;list=PLgWHRReJ5o_K35M3Fme9fkqjIij0WMO2SA</a>
4	Kani's, moment distribution	<a href="https://www.youtube.com/watch?v=aP2uuW2L3xk&amp;list=PLgWHRReJ5o_K0AIVa9GdfhbZdJb96uNZyD">https://www.youtube.com/watch?v=aP2uuW2L3xk&amp;list=PLgWHRReJ5o_K0AIVa9GdfhbZdJb96uNZyD</a>
5	Stiffness matrix method	<a href="https://www.youtube.com/watch?v=Y6QOqjAFI20&amp;list=PLgWHRReJ5o_K2njIjHWdK_0gOVOe068k1e">https://www.youtube.com/watch?v=Y6QOqjAFI20&amp;list=PLgWHRReJ5o_K2njIjHWdK_0gOVOe068k1e</a>
<b>Course: CE404UB (Professional Elective –III Advanced Reinforced Cement Concrete)</b>		

1	<b>Design of Combined footing</b>	<p><a href="https://www.youtube.com/watch?v=r_OCzx_IzV8&amp;list=PLgWHRReJ5o_K1Vd_eeeWw7X76OPys6KPSH">https://www.youtube.com/watch?v=r_OCzx_IzV8&amp;list=PLgWHRReJ5o_K1Vd_eeeWw7X76OPys6KPSH</a></p> <p><a href="https://www.youtube.com/watch?v=A-wPCbUiANo&amp;list=PLgWHRReJ5o_K1Vd_eeeWw7X76OPys6KPSH&amp;index=2">https://www.youtube.com/watch?v=A-wPCbUiANo&amp;list=PLgWHRReJ5o_K1Vd_eeeWw7X76OPys6KPSH&amp;index=2</a></p> <p><a href="https://www.youtube.com/watch?v=y263aNLXIRE&amp;list=PLgWHRReJ5o_K1Vd_eeeWw7X76OPys6KPSH&amp;index=3">https://www.youtube.com/watch?v=y263aNLXIRE&amp;list=PLgWHRReJ5o_K1Vd_eeeWw7X76OPys6KPSH&amp;index=3</a></p> <p><a href="https://www.youtube.com/watch?v=pb-yfJPUujE&amp;list=PLgWHRReJ5o_K1Vd_eeeWw7X76OPys6KPSH&amp;index=4">https://www.youtube.com/watch?v=pb-yfJPUujE&amp;list=PLgWHRReJ5o_K1Vd_eeeWw7X76OPys6KPSH&amp;index=4</a></p> <p><a href="https://www.youtube.com/watch?v=pb-yfJPUujE&amp;list=PLgWHRReJ5o_K1Vd_eeeWw7X76OPys6KPSH&amp;index=4">https://www.youtube.com/watch?v=pb-yfJPUujE&amp;list=PLgWHRReJ5o_K1Vd_eeeWw7X76OPys6KPSH&amp;index=4</a></p> <p><a href="https://www.youtube.com/watch?v=p_OIyV76k3c&amp;list=PLgWHRReJ5o_K1Vd_eeeWw7X76OPys6KPSH&amp;index=5">https://www.youtube.com/watch?v=p_OIyV76k3c&amp;list=PLgWHRReJ5o_K1Vd_eeeWw7X76OPys6KPSH&amp;index=5</a></p> <p><a href="https://www.youtube.com/watch?v=N74E1hBai1M&amp;list=PLgWHRReJ5o_K1Vd_eeeWw7X76OPys6KPSH&amp;index=6">https://www.youtube.com/watch?v=N74E1hBai1M&amp;list=PLgWHRReJ5o_K1Vd_eeeWw7X76OPys6KPSH&amp;index=6</a></p>
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2

**Design of Flat Slab**

[https://www.youtube.com/watch?v=CUZk45jLtkw&list=PLHUa7iJncyO28yRplzSCXd6\\_Z-uvd\\_nX&index=9&pp=gAQBiAQB](https://www.youtube.com/watch?v=CUZk45jLtkw&list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&index=9&pp=gAQBiAQB)

[https://www.youtube.com/watch?v=R1e-8DleKFA&list=PLHUa7iJncyO28yRplzSCXd6\\_Z-uvd\\_nX&index=10&pp=gAQBiAQB](https://www.youtube.com/watch?v=R1e-8DleKFA&list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&index=10&pp=gAQBiAQB)

[https://www.youtube.com/watch?v=mGQLCCN8AuQ&list=PLHUa7iJncyO28yRplzSCXd6\\_Z-uvd\\_nX&index=11&pp=gAQBiAQB](https://www.youtube.com/watch?v=mGQLCCN8AuQ&list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&index=11&pp=gAQBiAQB)

[https://www.youtube.com/watch?v=CFQSwTRe25c&list=PLHUa7iJncyO28yRplzSCXd6\\_Z-uvd\\_nX&index=12&pp=gAQBiAQB](https://www.youtube.com/watch?v=CFQSwTRe25c&list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&index=12&pp=gAQBiAQB)

[https://www.youtube.com/watch?v=OLnYAoaujE4&list=PLHUa7iJncyO28yRplzSCXd6\\_Z-uvd\\_nX&index=13&pp=gAQBiAQB](https://www.youtube.com/watch?v=OLnYAoaujE4&list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&index=13&pp=gAQBiAQB)

[https://www.youtube.com/watch?v=v0y8uc3syFQ&list=PLHUa7iJncyO28yRplzSCXd6\\_Z-uvd\\_nX&index=14&pp=gAQBiAQB](https://www.youtube.com/watch?v=v0y8uc3syFQ&list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&index=14&pp=gAQBiAQB)

[https://www.youtube.com/watch?v=y04Ta1\\_t81M&list=PLHUa7iJncyO28yRplzSCXd6\\_Z-uvd\\_nX&index=15](https://www.youtube.com/watch?v=y04Ta1_t81M&list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&index=15)

[https://www.youtube.com/watch?v=e4c2bRXk2YI&list=PLHUa7iJncyO28yRplzSCXd6\\_Z-uvd\\_nX&index=16](https://www.youtube.com/watch?v=e4c2bRXk2YI&list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&index=16)

[https://www.youtube.com/watch?v=hR95DG1881U&list=PLHUa7iJncyO28yRplzSCXd6\\_Z-uvd\\_nX&index=17](https://www.youtube.com/watch?v=hR95DG1881U&list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&index=17)



3

**RETAIINING WALLS**

[https://www.youtube.com/watch?v=Q4vMrRypApM&list=PLHUa7iJncyO28yRplzSCXd6\\_Z-uvd\\_nX&index=18](https://www.youtube.com/watch?v=Q4vMrRypApM&list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&index=18)

[https://www.youtube.com/watch?v=XsYeMM9Rtqo&list=PLHUa7iJncyO28yRplzSCXd6\\_Z-uvd\\_nX&index=20](https://www.youtube.com/watch?v=XsYeMM9Rtqo&list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&index=20)

[https://www.youtube.com/watch?v=Kiece700v\\_c&list=PLHUa7iJncyO28yRplzSCXd6\\_Z-uvd\\_nX&index=21](https://www.youtube.com/watch?v=Kiece700v_c&list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&index=21)

[https://www.youtube.com/watch?v=tmhqIDuh53M&list=PLHUa7iJncyO28yRplzSCXd6\\_Z-uvd\\_nX&index=22](https://www.youtube.com/watch?v=tmhqIDuh53M&list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&index=22)

[https://www.youtube.com/watch?v=PioF9QB3ysQ&list=PLHUa7iJncyO28yRplzSCXd6\\_Z-uvd\\_nX&index=23](https://www.youtube.com/watch?v=PioF9QB3ysQ&list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&index=23)

[https://www.youtube.com/watch?v=TmzNh\\_a7vX0&list=PLHUa7iJncyO28yRplzSCXd6\\_Z-uvd\\_nX&index=24](https://www.youtube.com/watch?v=TmzNh_a7vX0&list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&index=24)

[https://www.youtube.com/watch?v=7eQIj\\_U7Ac0&list=PLHUa7iJncyO28yRplzSCXd6\\_Z-uvd\\_nX&index=25](https://www.youtube.com/watch?v=7eQIj_U7Ac0&list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&index=25)

[https://www.youtube.com/watch?v=fu8t9hPeRhs&list=PLHUa7iJncyO28yRplzSCXd6\\_Z-uvd\\_nX&index=26](https://www.youtube.com/watch?v=fu8t9hPeRhs&list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&index=26)

4	<b>Design of Water tank with flexible base / fixed base</b>	<p><a href="https://www.youtube.com/watch?v=fu8t9hPeRhs&amp;list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&amp;index=26">https://www.youtube.com/watch?v=fu8t9hPeRhs&amp;list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&amp;index=26</a></p> <p><a href="https://www.youtube.com/watch?v=fu8t9hPeRhs&amp;list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&amp;index=26">https://www.youtube.com/watch?v=fu8t9hPeRhs&amp;list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&amp;index=26</a></p> <p><a href="https://www.youtube.com/watch?v=3QW5fbCpUiE&amp;list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&amp;index=27">https://www.youtube.com/watch?v=3QW5fbCpUiE&amp;list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&amp;index=27</a></p> <p><a href="https://www.youtube.com/watch?v=AwGa877Wnaw&amp;list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&amp;index=28">https://www.youtube.com/watch?v=AwGa877Wnaw&amp;list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&amp;index=28</a></p> <p><a href="https://www.youtube.com/watch?v=MnGwG1YEK1A&amp;list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&amp;index=29">https://www.youtube.com/watch?v=MnGwG1YEK1A&amp;list=PLHUa7iJncyO28yRplzSCXd6_Z-uvd_nX&amp;index=29</a></p>
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